5.6 Rates of Change in Rational Functions

YOU WILL NEED

- graphing calculator or graphing software

GOAL

Determine average rates of change, and estimate instantaneous rates of change for rational functions.

LEARN ABOUT the Math

The instantaneous rate of change at a point on a revenue function is called the *marginal revenue*. It is a measure of the estimated additional revenue from selling one more item.

For example, the demand equation for a toothbrush is $p(x) = \frac{5}{2 + x}$, where x is the number of toothbrushes sold, in thousands, and p is the price, in dollars.

? What is the marginal revenue when 1500 toothbrushes are sold? When is the marginal revenue the greatest? When is it the least?

EXAMPLE 1 Selecting a strategy to determine instantaneous rates of change

Determine the marginal revenue when 1500 toothbrushes are sold and when it is the greatest and the least.

Solution A: Calculating the average rate of change by squeezing centred intervals around $x = 1.5$

$$\text{Revenue } R(x) = xp(x) = \frac{5x}{2 + x}$$

Revenue = Number of items sold × Price

The average rate of change close to $x = 1.5$ is shown in the following table.

Centred Intervals	Average Rate of Change $\frac{R(x_2) - R(x_1)}{x_2 - x_1}$
$1.4 \le x \le 1.6$	0.817
$1.45 \le x \le 1.55$	0.816
$1.49 \le x \le 1.51$	0.816
$1.499 \le x \le 1.501$	0.816

x is measured in thousands, so when 1500 toothbrushes are sold, $x = 1.5$.

The average rate of change from $(x_1, R(x_1))$ to $(x_2, R(x_2))$ is the slope of the secant that joins each pair of endpoints.

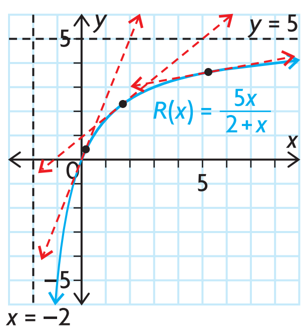
The average rate of change approaches 0.816. The marginal revenue when 1500 toothbrushes are sold is \$0.82 per toothbrush.

When x_1 and x_2 are very close to each other, the slope of the secant is approximately the same as the slope of the tangent. The slopes of the secants near the point where $x = 1.5$ approach 0.816.

Sketch the graph of $R(x) = \frac{5x}{2 + x}$.

The graph starts at (0, 0) and has a horizontal asymptote at $y = 5$.

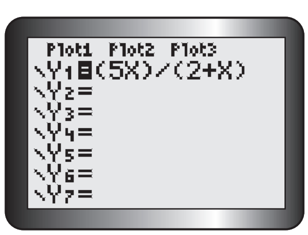
The vertical asymptote at $x = -2$ is not in the domain of $R(x)$, since $x \geq 0$, so it can be ignored.



The marginal revenue is the greatest when $x = 0$ and then decreases from there, approaching zero, as the graph gets closer to the horizontal asymptote.

In the context of the problem, x and $R(x)$ have only positive values. Examine the slope of the tangent lines at various points along the domain of the revenue graph. The slope is the greatest at the beginning of the graph and then decreases as x increases.

Solution B: Using the difference quotient and graphing technology to analyze the revenue function



Enter the revenue function into a graphing calculator.

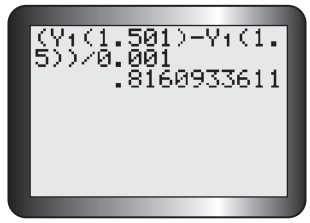
$$\text{Average rate of change} = \frac{R(a + h) - R(a)}{h}$$

Let $h = 0.001$

$$= \frac{R(1.5 + 0.001) - R(1.5)}{0.001}$$

$$= \frac{R(1.501) - R(1.5)}{0.001}$$

Use the difference quotient and a very small value for h, where $a = 1.5$, to estimate the instantaneous rate of change in revenue when 1500 toothbrushes are sold.

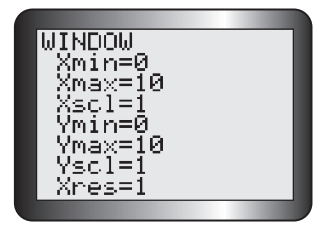


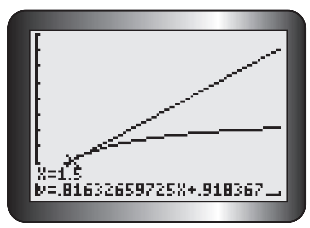
Enter the rate of change expression into the graphing calculator to determine its value, using the equation entered into Y1.

The average rate of change is about 0.816. The marginal revenue when 1500 toothbrushes are sold is \$0.82 per toothbrush.

To verify, graph the revenue function $R(x) = xp(x) = \frac{5x}{2 + x}$ and draw a tangent line at $x = 1.5$.

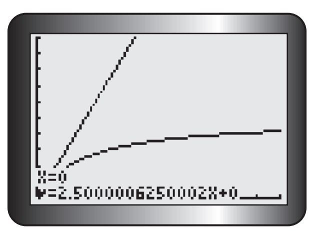
Since x and $R(x)$ only have positive values, graph the function in the first quadrant.



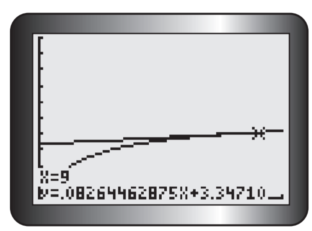


Use the DRAW feature of the graphing calculator to draw a tangent line where $x = 1.5$.

When 1500 toothbrushes are sold, the marginal revenue is \$0.82 per toothbrush.



The marginal revenue is the greatest when $x = 0$.



The marginal revenue decreases to very small values as x increases.

The tangent lines to this curve are steepest at the beginning of the curve. Their slopes decrease as x increases.

Reflecting

A. In Solution A, how were average rates of change used to estimate the instantaneous rate of change at a point?

B. In Solutions A and B, how were graphs used to estimate the instantaneous rate of change at a point?

C. In each solution, how was it determined where the marginal revenue was the greatest? Why was it not possible to determine the least marginal revenue?

D. What are the advantages and disadvantages of each method to determine the instantaneous rate of change?

APPLY the Math

EXAMPLE 2 Connecting the instantaneous rate of change to the slope of a tangent

a) Estimate the slope of the tangent to the graph of $f(x) = \frac{x}{x+3}$ at the point where $x = -5$.

b) Why can there not be a tangent line where $x = -3$?

Solution

a) $f(x) = \dfrac{x}{x+3}$

average rate of change $= \dfrac{f(a+h) - f(a)}{h}$

Let $h = 0.001$

$$= \frac{f(-5 + 0.001) - f(-5)}{0.001}$$

$$= \frac{f(-4.999) - f(-5)}{0.001}$$

$$= \frac{\left(\frac{-4.999}{-4.999 + 3}\right) - \left(\frac{-5}{-5 + 3}\right)}{0.001}$$

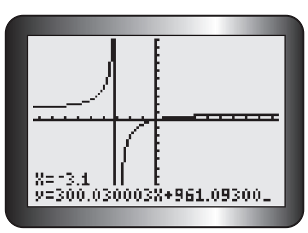
$$= \frac{2.500\,750\,375 - 2.5}{0.001}$$

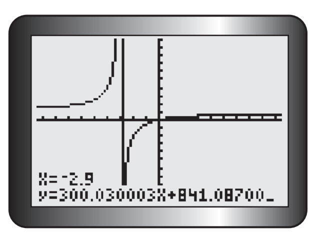
$$\doteq 0.7504$$

Use the difference quotient and a very small value for h close to $a = -5$ to estimate the slope of the tangent where $x = -5$.

The slope of the tangent at $x = -5$ is 0.75.

b) The value -3 is not in the domain of $f(x)$, so no tangent line is possible there. The graph of $f(x)$ has a vertical asymptote at $x = -3$.





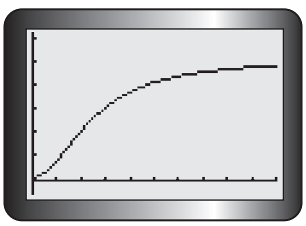
As x approaches -3 from the left and from the right, the tangent lines are very steep. The tangent lines approach a vertical line, but are never actually vertical. There is no point on the graph with an x-coordinate of -3, so there is no tangent line there.

EXAMPLE 3 Selecting a graphing strategy to solve a problem that involves average and instantaneous rates of change

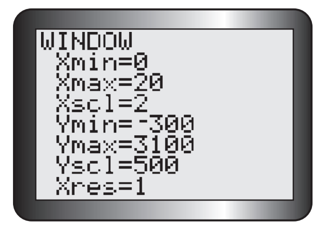
The snowshoe hare population in a newly created conservation area can be predicted over time by the model $p(t) = 50 + \frac{2500t^2}{25 + t^2}$, where p represents the population size and t is the time in years since the opening of the conservation area. Determine when the hare population will increase most rapidly, and estimate the instantaneous rate of change in population at this time.

Solution

Graph $p(t) = 50 + \frac{2500t^2}{25 + t^2}$ for $0 \leq t \leq 20$.

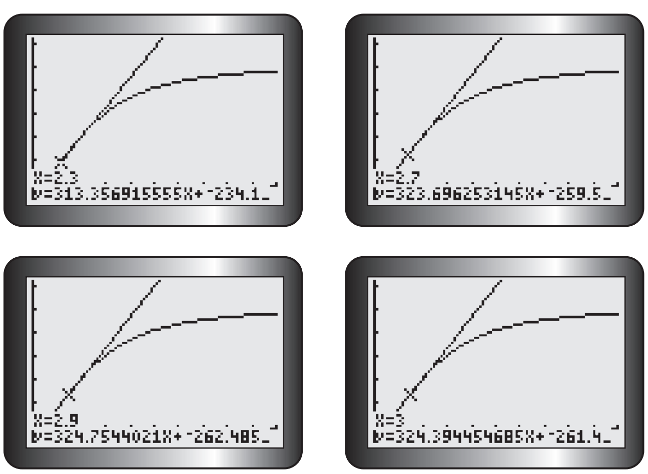


$p(t) \geq 0$, but you can set the minimum y-value to a negative number to allow some space for displayed values.



The slopes of the tangent lines increase slowly at the beginning of the graph. The slopes start to increase more rapidly around $t = 2$. They begin to decrease after $t = 3$.

Draw tangent lines between 2 and 3, and look for the tangent line that has the greatest slope.



The slopes of the tangent lines increase until 2.9, and then decrease. The tangent line at $t = 2.9$ has the greatest slope.

The average rate of change is greatest when t is close to 2.9. The hare population will increase most rapidly about 2 years and 11 months after the conservation area is opened. The instantaneous rate of change in population at this time is approximately 325 hares per year.

0.9×12 months is approximately 11 months.

In Summary

Key Ideas

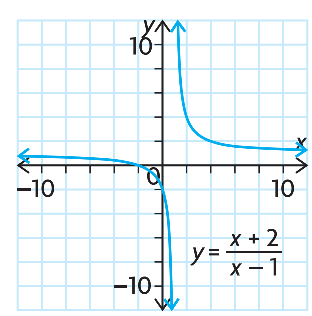
- The methods that were previously used to calculate the average rate of change and estimate the instantaneous rate of change can be used for rational functions.
- You cannot determine the average and instantaneous rates of change of a rational function at a point where the graph is discontinuous (that is, where there is a hole or a vertical asymptote).

Need to Know

- The average rate of change of a rational function, $y = f(x)$, on the interval from $x_1 \leq x \leq x_2$ is $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$. Graphically, this is equivalent to the slope of the secant line that passes through the points (x_1, y_1) and (x_2, y_2) on the graph of $y = f(x)$.
- The instantaneous rate of change of a rational function, $y = f(x)$, at $x = a$ can be approximated using the difference quotient $\frac{f(a + h) - f(a)}{h}$ and a very small value of h. Graphically, this is equivalent to estimating the slope of the tangent line that passes through the point $(a, f(a))$ on the graph of $y = f(x)$.
- The instantaneous rate of change at a vertical asymptote is undefined. The instantaneous rates of change at points that are approaching a vertical asymptote become very large positive or very large negative values. The instantaneous rate of change near a horizontal asymptote approaches zero.

CHECK Your Understanding

1. The graph of a rational function is shown.



a) Determine the average rate of change of the function over the interval $2 \leq x \leq 7$.

b) Copy the graph, and draw a tangent line at the point where $x = 2$. Determine the slope of the tangent line to estimate the instantaneous rate of change at this point.

2. Estimate the instantaneous rate of change of the function in question 1 at $x = 2$ by determining the slope of a secant line from the point where $x = 2$ to the point where $x = 2.01$. Compare your answer with your answer for question 1, part b).

3. Use graphing technology to estimate the instantaneous rate of change of the function in question 1 at $x = 2$.

PRACTISING

4. Estimate the instantaneous rate of change of $f(x) = \frac{x}{x - 4}$ at the point $(2, -1)$.

5. **K** Select a strategy to estimate the instantaneous rate of change of each function at the given point.

a) $y = \dfrac{1}{25 - x}$, where $x = 13$

b) $y = \dfrac{17x + 3}{x^2 + 6}$, where $x = -5$

c) $y = \dfrac{x + 3}{x - 2}$, where $x = 4$

d) $y = \dfrac{-3x^2 + 5x + 6}{x + 6}$, where $x = -3$

6. Determine the slope of the line that is tangent to the graph of each function at the given point. Then determine the value of x at which there is no tangent line.

a) $f(x) = \dfrac{-5x}{2x + 3}$, where $x = 2$

b) $f(x) = \dfrac{x - 6}{x + 5}$, where $x = -7$

c) $f(x) = \dfrac{2x^2 - 6x}{3x + 5}$, where $x = -2$

d) $f(x) = \dfrac{5}{x - 6}$, where $x = 4$

7. **A** When polluted water begins to flow into an unpolluted pond, the concentration of pollutant, c, in the pond at t minutes is modelled by $c(t) = \dfrac{27t}{10\,000 + 3t}$, where c is measured in kilograms per cubic metre. Determine the rate at which the concentration is changing after

a) 1 h **b)** one week

8. The demand function for snack cakes at a large bakery is given by the function $p(x) = \dfrac{15}{2x^2 + 11x + 5}$. The x-units are given in thousands of cakes, and the price per snack cake, $p(x)$, is in dollars.

a) Find the revenue function for the cakes.

b) Estimate the marginal revenue for $x = 0.75$. What is the marginal revenue for $x = 2.00$?

9. At a small clothing company, the estimated average cost function for producing a new line of T-shirts is $C(x) = \dfrac{x^2 - 4x + 20}{x}$, where x is the number of T-shirts produced, in thousands. $C(x)$ is measured in dollars.

a) Calculate the average cost of a T-shirt at a production level of 3000 pairs.

b) Estimate the rate at which the average cost is changing at a production level of 3000 T-shirts.

10. Suppose that the number of houses in a new subdivision after t months of development is modelled by $N(t) = \frac{100t^3}{100 + t^3}$, where N is the number of houses and $0 \leq t \leq 12$.

a) Calculate the average rate of change in the number of houses built over the first 6 months.

b) Calculate the instantaneous rate of change in the number of houses built at the end of the first year.

c) Graph the function using a graphing calculator. Discuss what happens to the rate at which houses were built in this subdivision during the first year of development.

11. **T** Given the function $f(x) = \frac{x - 2}{x - 5}$, determine an interval and a point where the average rate of change and the instantaneous rate of change are equal.

12. **C** **a)** The position of an object that is moving along a straight line at t seconds is given by $s(t) = \frac{3t}{t + 4}$, where s is measured in metres. Explain how you would determine the average rate of change of $s(t)$ over the first 6 s.

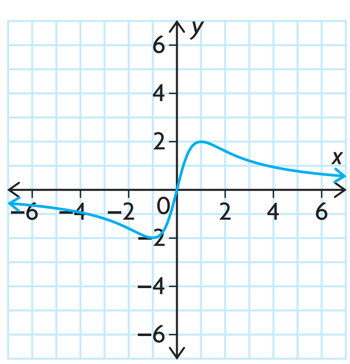
b) What does the average rate of change mean in this context?

c) Compare two ways that you could determine the instantaneous rate of change when $t = 6$. Which method is easier? Explain. Which method is more accurate? Explain.

d) What does the instantaneous rate of change mean in this context?

Extending

13. The graph of the rational function $f(x) = \frac{4x}{x^2 + 1}$ has been given the name Newton's Serpentine. Determine the equations for the tangents at the points where $x = -\sqrt{3}$, 0, and $\sqrt{3}$.



14. Determine the instantaneous rate of change of Newton's Serpentine at points around the point (0, 0). Then determine the instantaneous rate of change of this instantaneous rate of change.

5 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

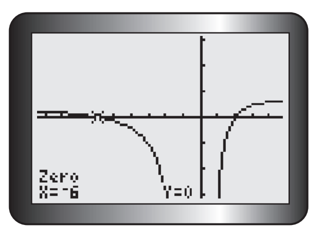
- See Lesson 5.4, Examples 1, 2, and 3.
- Try Chapter Review Questions 7, 8, and 9.

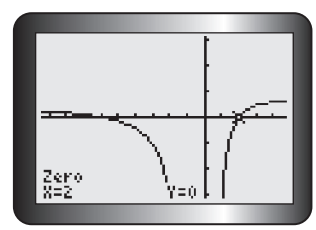
Q: How do you solve and verify a rational equation such as $\frac{3x - 8}{2x - 1} = \frac{x - 4}{x + 1}$?

A: You can solve a simple rational equation algebraically by multiplying each term in the equation by the lowest common denominator and then solving the resulting polynomial equation.

For example, to solve $\frac{3x - 8}{2x - 1} = \frac{x - 4}{x + 1}$, multiply the equation by $(2x - 1)(x + 1)$, where $x \neq -1$ or $\frac{1}{2}$. Then solve the resulting polynomial equation.

To verify your solutions, you can graph the corresponding function, $f(x) = \frac{3x - 8}{2x - 1} - \frac{x - 4}{x + 1}$, using graphing technology and determine the zeros of f.





The zeros are -6 and 2, so the solution to the equation is $x = -6$ or 2.

Study *Aid*

- See Lesson 5.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 10 and 11.

Q: How do you solve a rational inequality, such as $\frac{x - 2}{x + 1} > \frac{x - 6}{x - 2}$?

A1: You can solve a rational inequality algebraically by creating and solving an equivalent linear or polynomial inequality with zero on one side. For factorable polynomial inequalities of degree 2 or more, use a table to identify the positive/negative intervals created by the zeros and vertical asymptotes of the rational expression.

A2: You can use graphing technology to graph the functions on both sides of the inequality, determine their intersection and the locations of all vertical asymptotes, and then note the intervals of x that satisfy the inequality.

Q: How do you determine the average or instantaneous rate of change of a rational function?

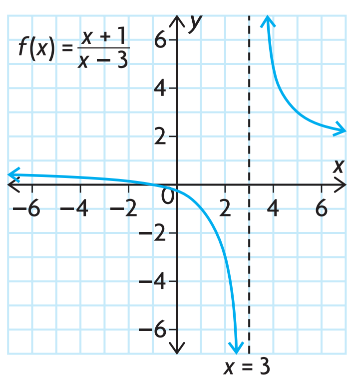
A: You can determine average and instantaneous rates of change of a rational function at points within the domain of the function using the same methods that are used for polynomial functions.

Study Aid

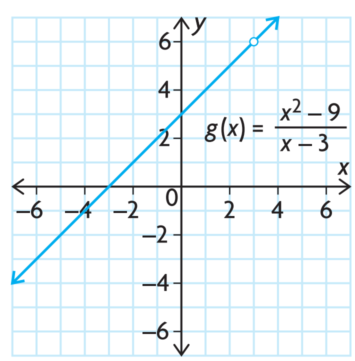
- See Lesson 5.6, Examples 1, 2, and 3.
- Try Chapter Review Questions 12, 13, and 14.

Q: When is it not possible to determine the average or instantaneous rate of change of a rational function?

A: You cannot determine the average and instantaneous rates of change of a rational function at a point where the graph has a hole or a vertical asymptote. You can only calculate the instantaneous rate of change at a point where the rational function is defined and where a tangent line can be drawn. A rational function is not defined at a point where there is a hole or a vertical asymptote. For example, $f(x) = \frac{x+1}{x-3}$ and $g(x) = \frac{x^2-9}{x-3}$ are rational functions that are not defined at $x = 3$.



The graph of $f(x)$ has a vertical asymptote at $x = 3$.



The graph of $g(x)$ has a hole at $x = 3$.

You cannot draw a tangent line on either graph at $x = 3$, so you cannot determine an instantaneous rate of change at this point.

PRACTICE Questions

Lesson 5.1

1. For each function, determine the domain and range, intercepts, positive/negative intervals, and increasing and decreasing intervals. Use this information to sketch a graph of the reciprocal function.

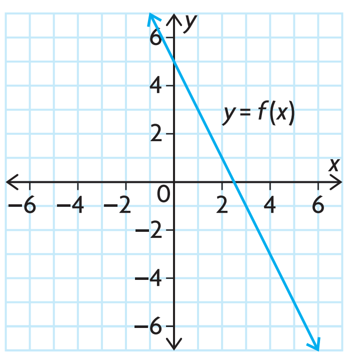
a) $f(x) = 3x + 2$

b) $f(x) = 2x^2 + 7x - 4$

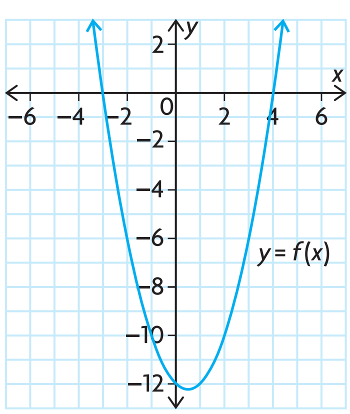
c) $f(x) = 2x^2 + 2$

2. Given the graphs of $f(x)$ below, sketch the graphs of $y = \frac{1}{f(x)}$.

a)



b)



Lesson 5.2

3. For each function, determine the equations of any vertical asymptotes, the locations of any holes, and the existence of any horizontal or oblique asymptotes.

a) $y = \dfrac{1}{x + 17}$

b) $y = \dfrac{2x}{5x + 3}$

c) $y = \dfrac{3x + 33}{-4x^2 - 42x + 22}$

d) $y = \dfrac{3x^2 - 2}{x - 1}$

Lesson 5.3

4. The population of locusts in a Prairie a town over the last 50 years is modelled by the function $f(x) = \frac{75x}{x^2 + 3x + 2}$. The locust population is given in hundreds of thousands. Describe the locust population in the town over time, where x is time in years.

5. For each function, determine the domain, intercepts, asymptotes, and positive/negative intervals. Use these characteristics to sketch the graph of the function. Then describe where the function is increasing or decreasing.

a) $f(x) = \dfrac{2}{x + 5}$

b) $f(x) = \dfrac{4x - 8}{x - 2}$

c) $f(x) = \dfrac{x - 6}{3x - 18}$

d) $f(x) = \dfrac{4x}{2x + 1}$

6. Describe how you can determine the behaviour of the values of a rational function on either side of a vertical asymptote.

Lesson 5.4

7. Solve each equation algebraically, and verify your solution using a graphing calculator.

a) $\frac{x-6}{x+2} = 0$

b) $15x + 7 = \frac{2}{x}$

c) $\frac{2x}{x-12} = \frac{-2}{x+3}$

d) $\frac{x+3}{-4x} = \frac{x-1}{-4}$

8. A group of students have volunteered for the student council car wash. Janet can wash a car in m minutes. Rodriguez can wash a car in $m - 5$ minutes, while Nick needs the same amount of time as Janet. If they all work together, they can wash a car in about 3.23 minutes. How long does Janet take to wash a car?

9. The concentration of a toxic chemical in a spring-fed lake is given by the equation $c(x) = \frac{50x}{x^2 + 3x + 6}$, where c is given in grams per litre and x is the time in days. Determine when the concentration of the chemical is 6.16 g/L.

Lesson 5.5

10. Use an algebraic process to find the solution set of each inequality. Verify your answers using graphing technology.

a) $-x + 5 < \frac{1}{x+3}$

b) $\frac{55}{x+16} > -x$

c) $\frac{2x}{3x+4} > \frac{x}{x+1}$

d) $\frac{x}{6x-9} \leq \frac{1}{x}$

11. A biologist predicted that the population of tadpoles in a pond could be modelled by the function $f(t) = \frac{40t}{t^2+1}$, where t is given in days. The function that actually models the tadpole population is $g(t) = \frac{45t}{t^2 + 8t + 7}$. Determine where $g(t) > f(t)$.

Lesson 5.6

12. Estimate the slope of the line that is tangent to each function at the given point. At what point(s) is it not possible to draw a tangent line?

a) $f(x) = \frac{x+3}{x-3}$, where $x = 4$

b) $f(x) = \frac{2x-1}{x^2+3x+2}$, where $x = 1$

13. The concentration, c, of a drug in the bloodstream t hours after the drug was taken orally is given by $c(t) = \frac{5t}{t^2+7}$, where c is measured in milligrams per litre.

a) Calculate the average rate of change in the drug's concentration during the first 2 h since ingestion.

b) Estimate the rate at which the concentration of the drug is changing after exactly 3 h.

c) Graph $c(t)$ on a graphing calculator. When is the concentration of the drug increasing the fastest in the bloodstream? Explain.

14. Given the function $f(x) = \frac{2x}{x-4}$, determine the coordinates of a point on $f(x)$ where the slope of the tangent line equals the slope of the secant line that passes through $A(5, 10)$ and $B(8, 4)$.

15. Describe what happens to the slope of a tangent line on the graph of a rational function as the x-coordinate of the point of tangency

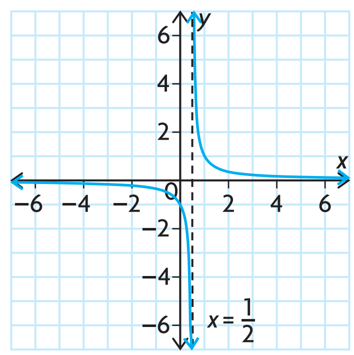
a) gets closer and closer to the vertical asymptote.

b) grows larger in both the positive and negative direction.

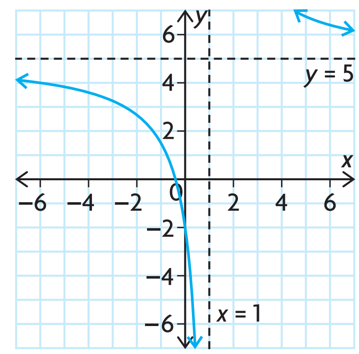
5 Chapter Self-Test

1. Match each graph with the equation of its corresponding function.

a)



b)



A $y = \dfrac{5x + 2}{x - 1}$ **B** $y = \dfrac{1}{2x - 1}$

2. Suppose that n is a constant and that $f(x)$ is a linear or quadratic function defined when $x = n$. Complete the following sentences.

a) If $f(n)$ is large, then $\dfrac{1}{f(n)}$ is....

b) If $f(n)$ is small, then $\dfrac{1}{f(n)}$ is....

c) If $f(n) = 0$, then $\dfrac{1}{f(n)}$ is....

d) If $f(n)$ is positive, then $\dfrac{1}{f(n)}$ is....

3. Without using graphing technology, sketch the graph of $y = \dfrac{2x + 6}{x - 2}$.

4. A company purchases x kilograms of steel for \$2249.52. The company processes the steel and turns it into parts that can be used in other factories. After this process, the total mass of the steel has dropped by 25 kg (due to trimmings, scrap, and so on), but the value of the steel has increased to \$10 838.52. The company has made a profit of \$2/kg. What was the original mass of the steel? What is the original cost per kilogram?

5. Select a strategy to solve each of the following.

a) $\dfrac{-x}{x - 1} = \dfrac{-3}{x + 7}$ **b)** $\dfrac{2}{x + 5} > \dfrac{3x}{x + 10}$

6. If you are given the equation of a rational function of the form $f(x) = \dfrac{ax + b}{cx + d}$, explain

a) how you can determine the equations of all vertical and horizontal asymptotes without graphing the function

b) when this type of function would have a hole instead of a vertical asymptote

5 Chapter Task

A New School

Researchers at a school board have developed models to predict population changes in the three areas they service. The models are $A(t) = \frac{360}{t + 6}$ for area A, $B(t) = \frac{30t}{t + 1}$ for area B, and $C(t) = \frac{50}{41 - 2t}$ for area C, where the population is measured in thousands and t is the time, in years, since 2007. The existing schools are full, and the board has agreed that a new school should be built.

? In which area should the new school be built, and when will the new school be needed?

A. Graph each population function for the 20 years following 2007. Use your graphs to describe the population trends in each area between 2007 and 2027.

B. Describe the intervals of increase or decrease for each function.

C. Determine which area will have the greatest population in 2010, 2017, 2022, and 2027.

D. Determine the intervals over which
- the population of area A is greater than the population of area B
- the population of area A is greater than the population of area C
- the population of area B is greater than the population of area C

E. Determine when the population of area B will be increasing most rapidly and when the population of area C will be increasing most rapidly.

F. What will happen to the population in each area over time?

G. Decide where and when the school should be built. Compile your results into a recommendation letter to the school board.

Task *Checklist*

✔ Did you show all your steps?

✔ Did you draw and label your graphs accurately?

✔ Did you support your choice of location for the school?

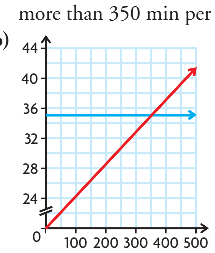
✔ Did you explain your thinking clearly?

6. **a)** Answers may vary. For example, $2x + 1 > 17$
b) Answers may vary. For example, $3x - 4 \geq -16$
c) Answers may vary. For example, $2x + 3 \leq -21$
d) Answers may vary. For example, $-19 < 2x - 1 < -3$

7. **a)** $x \in \left(\frac{25}{2}, \infty\right)$
b) $x \in \left[-\frac{23}{8}, \infty\right)$
c) $x \in (-\infty, 2)$
d) $x \in (-\infty, 3]$

8. **a)** $\{x \in \mathbf{R} \mid -2 < x < 4\}$
b) $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$
c) $\{x \in \mathbf{R} \mid -3 \leq x \leq 5\}$
d) $\{x \in \mathbf{R} \mid -6 < x < -2\}$

9. **a)** The second plan is better if one calls more than 350 min per month.
b)



10. **a)** $-1 < x < 2$
b) $x \leq -\frac{3}{2}$ or $x \geq 5$
c) $x < -\frac{5}{2}$ or $1 < x < 7$
d) $x \leq -4$ or $1 \leq x \leq 5$

11. negative when $x \in (0, 5)$, positive when $x \in (-\infty, -2), (-2, 0), (5, \infty)$

12. $x \leq -3.81$

13. between January 1993 and March 1994 and between October 1995 and October 1996

14. **a)** average $= 7$, instantaneous $\doteq 8$
b) average $= 13$, instantaneous $\doteq 15$
c) average $= 129$, instantaneous $\doteq 145$
d) average $= -464$, instantaneous $\doteq -485$

15. positive when $-1 < x < 1$, negative when $x < -1$ or $x > 1$, and zero at $x = -1, 1$

16. **a)** $t \doteq 2.2$ s
b) -11 m/s
c) about -22 m/s

17. **a)** about 57.002
b) about 56.998
c) Both approximate the instantaneous rate of change at $x = 3$.

18. **a)** male: $f(x) = 0.001x^3 - 0.162x^2 + 3.394x + 72.365$; female: $g(x) = 0.0002x^3 - 0.026x^2 + 1.801x + 14.369$
b) More females than males will have lung cancer in 2006.
c) The rate was changing faster for females, on average. Looking only at 1975 and 2000, the incidence among males increased only 5.5 per 100 000, while the incidence among females increased by 31.7.
d) Between 1995 and 2000, the incidence among males decreased by 6.1 while the incidence among females increased by 5.6. Since 1998 is about halfway between 1995 and 2000, an estimate for the instantaneous rate of change in 1998 is the average rate of change from 1995 to 2000. The two rates of change are about the same in magnitude, but the rate for females is positive, while the rate for males is negative.

Chapter Self-Test, p. 242

1. $1, \frac{3}{2}, -2$

2. **a)** positive when $x < -2$ and $0 < x < 2$, negative when $-2 < x < 0$ and $x > 2$, and zero at $-2, 0, 2$
b) positive when $-1 < x < 1$, negative when $x < -1$ or $1 < x$, and zero at $x = -1, 1$
c) -1

3. **a)** Cost with card: $50 + 5n$; Cost without card: $12n$
b) at least 8 pizzas

4. **a)** $x < \frac{1}{2}$
b) $-2 \leq x \leq 1$
c) $-2 < x < -1$ or $x > 5$
d) $x \geq -3$

5. **a)** 15 m
b) 4.6 s
c) -3 m/s

6. **a)** about 5 **b)** $(1, 3)$ **c)** $y = 5x - 2$

7. Since all the exponents are even and all the coefficients are positive, all values of the function are positive and greater than or equal to 4 for all real numbers x.

8. **a)** $\{x \in \mathbf{R} \mid -2 \leq x \leq 7\}$
b) $-2 < x < 7$

9. 2 cm by 2 cm by 15 cm

Chapter 5

Getting Started, pp. 246–247

1. **a)** $(x - 5)(x + 2)$
b) $3(x + 5)(x - 1)$
c) $(4x - 7)(4x + 7)$
d) $(3x - 2)(3x - 2)$
e) $(a - 3)(3a + 10)$
f) $(2x + 3y)(3x - 7y)$

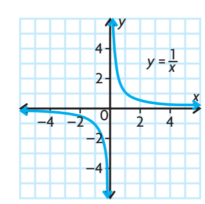
2. **a)** $3 - 2s$
b) $\frac{n^3}{3m}$, $m, n \neq 0$
c) $3x^2 - 4x - 1$, $x \neq 0$
d) $\frac{1}{5x - 2}$, $x \neq \frac{2}{5}$
e) $-\frac{x + 6}{3 + x}$, $x \neq -3, 3$
f) $\frac{a - b}{a - 3b}$, $a \neq -5b, \frac{3b}{2}$

3. **a)** $\frac{7}{15}$
b) $\frac{6}{x}$, $x \neq 0$
c) $\frac{-4x^2 + 20x - 6}{x - 3}$, $x \neq -2, 3$
d) $\frac{x^3 + 2x - 8x}{x^2 - 1}$, $x \neq -1, 0, 1, 3$

4. **a)** $1\frac{11}{21}$
b) $\frac{19x}{12}$
c) $\frac{4 + x}{x^2}$, $x \neq 0$
d) $\frac{3x - 6}{x^2 - 3x}$, $x \neq 0, 3$
e) $\frac{2x + 10 + y}{x^2 - 25}$, $x \neq 5, -5$
f) $\frac{-2a + 50}{(a + 3)(a - 5)(a + 3)}$, $x \neq -3, 4, 5$

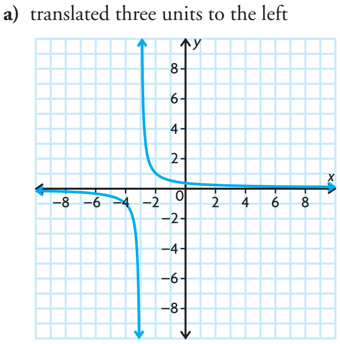
5. **a)** $x = 6$
b) $x = 2$
c) $x = 3$
d) $x = \frac{-12}{7}$

6.

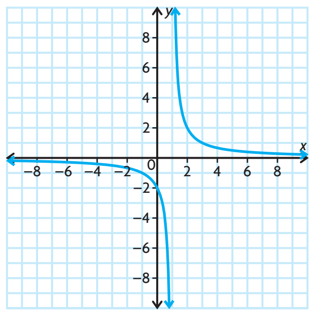


vertical: $x = 0$; horizontal: $y = 0$;
$D = \{x \in \mathbf{R} \mid x \neq 0\}$;
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

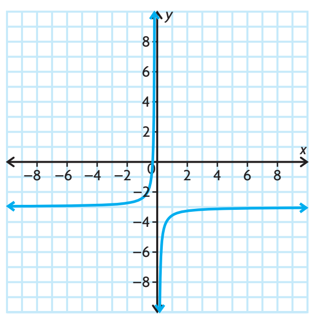
7. **a)** translated three units to the left



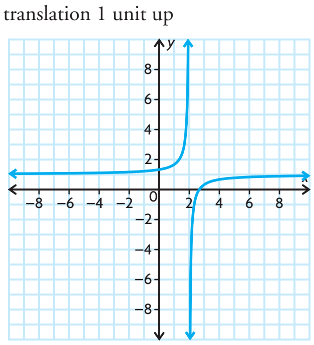
b) vertical stretch by a factor of 2 and a horizontal translation 1 unit to the right



c) reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, and a vertical translation 3 units down



d) reflection in the x-axis, vertical compression by a factor of $\frac{2}{3}$, horizontal translation 2 units right, and a vertical translation 1 unit up



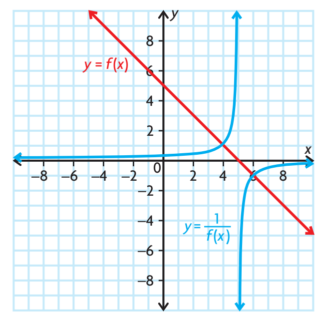
8. Factor the expressions in the numerator and the denominator. Simplify each expression as necessary. Multiply the first expression by the reciprocal of the second.
$\dfrac{-3(3y-2)}{2(3y+2)}$

Lesson 5.1, pp. 254–257

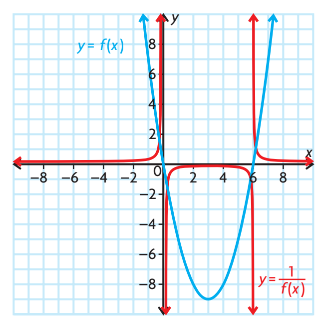
1. **a)** C; The reciprocal function is F.
b) A; The reciprocal function is E.
c) D; The reciprocal function is B.
d) F; The reciprocal function is C.
e) B; The reciprocal function is D.
f) E; The reciprocal function is A.

2. **a)** $x = 6$
b) $x = -\frac{4}{3}$
c) $x = 5$ and $x = -3$
d) $x = -\frac{5}{2}$ and $x = \frac{5}{2}$
e) no asymptotes
f) $x = -1.5$ and $x = -1$

3. **a)**



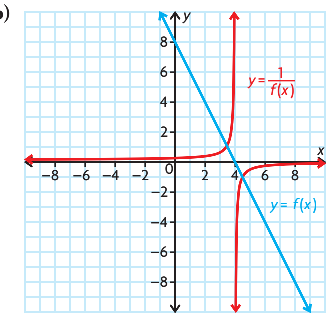
b)



4. **a)**

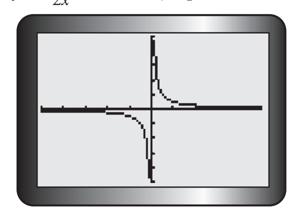
x	$f(x)$	$\frac{1}{f(x)}$
-4	16	$\frac{1}{16}$
-3	14	$\frac{1}{14}$
-2	12	$\frac{1}{12}$
-1	10	$\frac{1}{10}$
0	8	$\frac{1}{8}$
1	6	$\frac{1}{6}$
2	4	$\frac{1}{4}$
3	2	$\frac{1}{2}$
4	0	undefined
5	-2	$-\frac{1}{2}$
6	-4	$-\frac{1}{4}$
7	-6	$-\frac{1}{6}$

b)

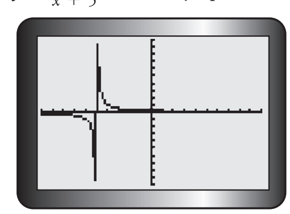


c) $f(x) = -2x + 8$, $y = \dfrac{1}{-2x+8}$

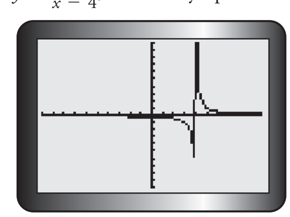
5. **a)** $y = \frac{1}{2x}$; vertical asymptote at $x = 0$



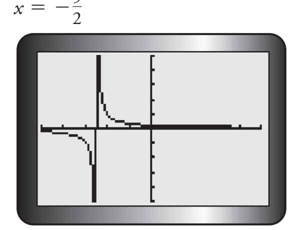
b) $y = \frac{1}{x+5}$; vertical asymptote at $x = -5$



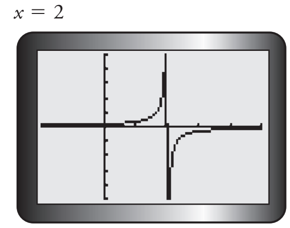
c) $y = \frac{1}{x-4}$; vertical asymptote at $x = 4$



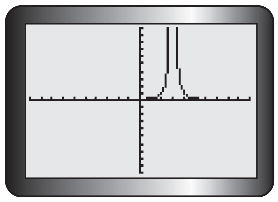
d) $y = \frac{1}{2x+5}$; vertical asymptote at $x = -\frac{5}{2}$



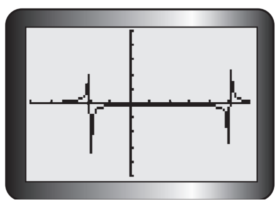
e) $y = \frac{1}{-3x+6}$; vertical asymptote at $x = 2$



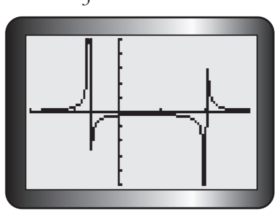
f) $y = \frac{1}{(x-3)^2}$; vertical asymptote at $x = 3$



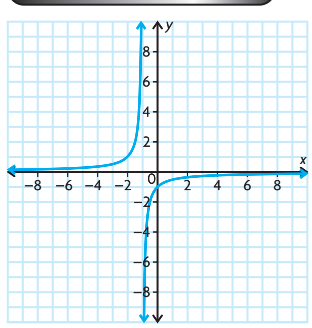
g) $y = \frac{1}{x^2 - 3x - 10}$; vertical asymptotes at $x = -2$ and $x = 5$



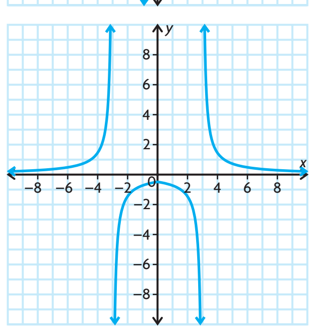
h) $y = \frac{1}{3x^2 - 4x - 4}$; vertical asymptotes at $x = -\frac{2}{3}$ and $x = 2$



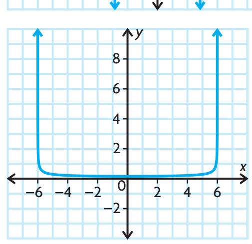
6. a)



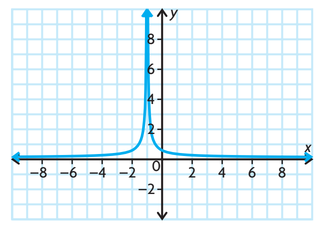
b)



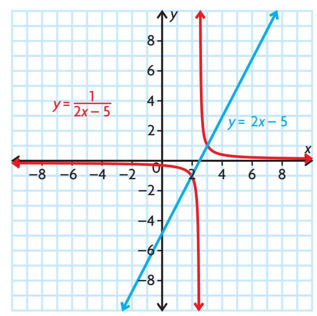
c)



d)

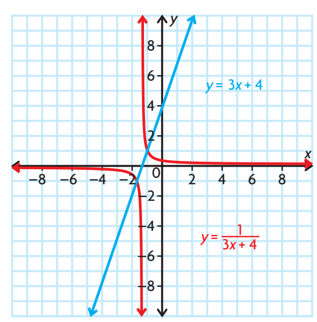


7. a)



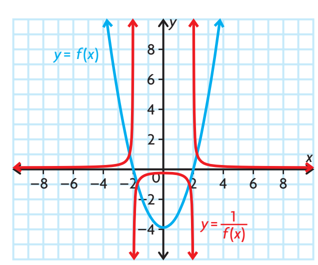
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

b)

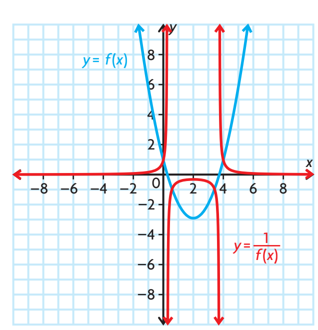


$D = \left\{x \in \mathbf{R} \mid x \neq -\frac{4}{3}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

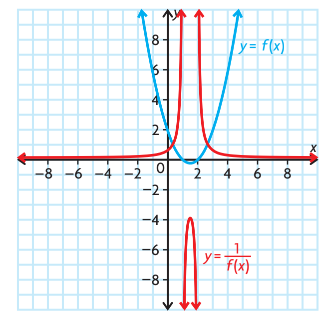
8. a)



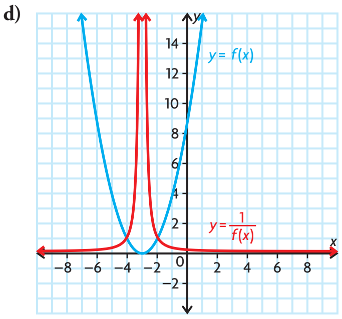
b)



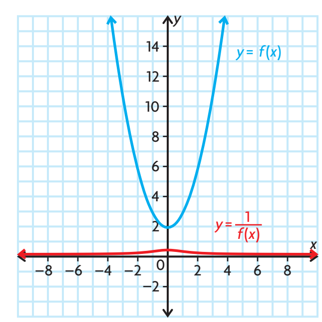
c)



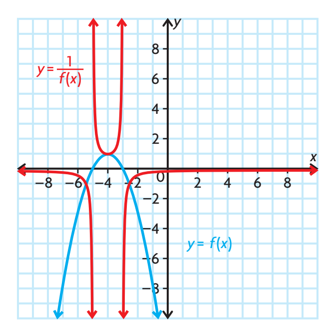
d)



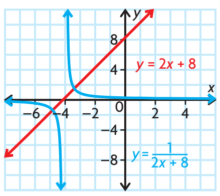
e)



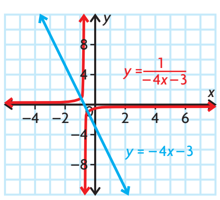
f)



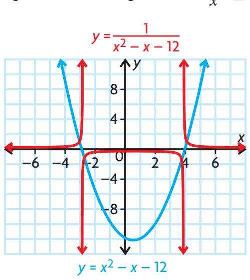
9. **a)** $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$
y-intercept $= 8$
x-intercept $= -4$
negative on $(-\infty, -4)$
positive on $(-4, -\infty)$
increasing on $(-\infty, \infty)$
equation of reciprocal $= \dfrac{1}{2x + 8}$



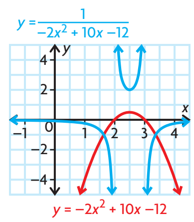
b) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$
y-intercept $= -3$
x-intercept $= -\frac{3}{4}$
positive on $\left(-\infty, -\frac{3}{4}\right)$
negative on $\left(-\frac{3}{4}, \infty\right)$
decreasing on $(-\infty, \infty)$
equation of reciprocal $= \dfrac{1}{-4x - 3}$



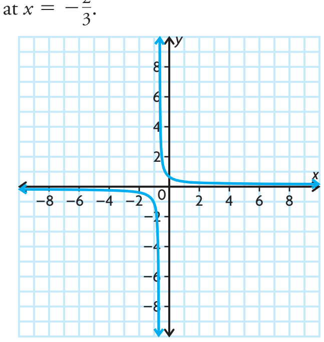
c) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid y \geq -12.25\}$
y-intercept $= 12$
x-intercepts $= 4, -3$
decreasing on $(-\infty, 0.5)$
increasing on $(0.5, \infty)$
positive on $(-\infty, -3)$ and $(4, \infty)$
negative on $(-3, 4)$
equation of reciprocal $= \dfrac{1}{x^2 - x - 12}$



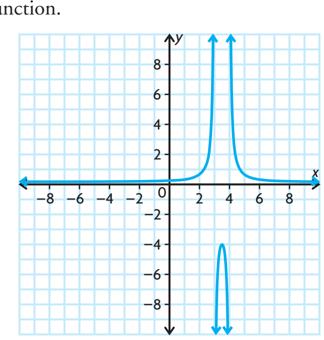
d) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} \mid y \leq 2.5\}$
y-intercept $= -12$
x-intercepts $= 3, 2$
increasing on $(-\infty, 2.5)$
decreasing on $(2.5, \infty)$
negative on $(-\infty, 2)$ and $(3, \infty)$
positive on $(2, 3)$
equation of reciprocal $= \dfrac{1}{-2x^2 + 10x - 12}$



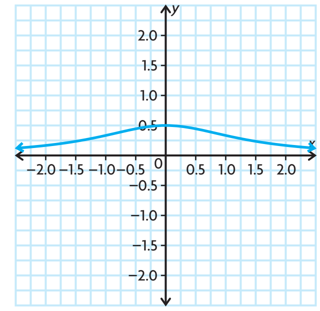
10. Answers may vary. For example, a reciprocal function creates a vertical asymptote when the denominator is equal to 0 for a specific value of x. Consider $\frac{1}{ax + b}$. For this expression, there is always some value of x that is $\frac{-b}{a}$ that will result in a vertical asymptote for the function. This is a graph of $y = \frac{1}{3x + 2}$ and the vertical asymptote is at $x = -\frac{2}{3}$.



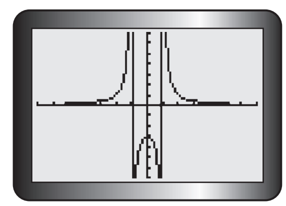
Consider the function $\frac{1}{(x - 3)(x - 4)}$. The graph of the quadratic function in the denominator crosses the x-axis at 3 and 4 and therefore will have vertical asymptotes at 3 and 4 in the graph of the reciprocal function.



However, a quadratic function, such as $x^2 + c$, which has no real zeros, will not have a vertical asymptote in the graph of its reciprocal function. For example, this is the graph of $y = \frac{1}{x^2 + 2}$.

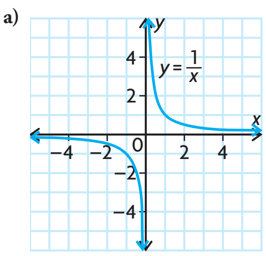


11. $y = \dfrac{3}{x^2 - 1}$



12. **a)** 500
b) $t = 2$
c) $t = 10\,000$
d) If you were to use a value of t that was less than one, the equation would tell you that the number of bacteria was increasing as opposed to decreasing. Also, after time $t = 10\,000$, the formula indicates that there is a smaller and smaller fraction of 1 bacteria left.
e) $D = \{x \in \mathbf{R} \mid 1 < x < 10\,000\}$,
$R = \{y \in \mathbf{R} \mid 1 < y < 10\,000\}$

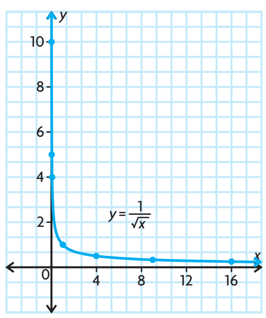
13. **a)**



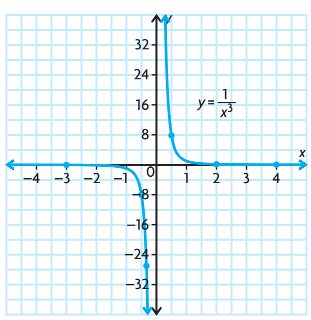
$D = \{x \in \mathbf{R} \mid x \neq -n\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
b) The vertical asymptote occurs at $x = -n$. Changes in n in the $f(x)$ family cause changes in the y-intercept—an increase in n causes the intercept to move up the y-axis and a decrease causes it to move down the y-axis. Changes in n in the $g(x)$ family cause changes in the vertical asymptote of the function—an increase in n causes the asymptote to move down the x-axis and a decrease in n causes it to move up the x-axis.
c) $x = 1 - n$ and $x = -1 - n$

14. Answers may vary. For example:
1) Determine the zero(s) of the function $f(x)$—these will be the asymptote(s) for the reciprocal function $g(x)$.
2) Determine where the function $f(x)$ is positive and where it is negative—the reciprocal function $g(x)$ will have the same characteristics.
3) Determine where the function $f(x)$ is increasing and where it is decreasing—the reciprocal function $g(x)$ will have opposite characteristics.

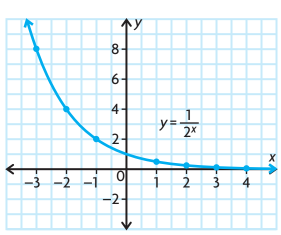
15. **a)**



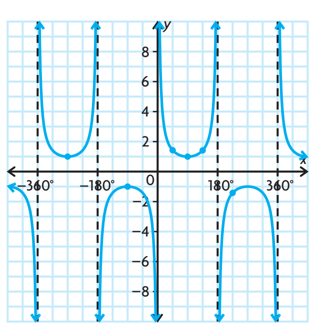
b)



c)



d)



16. $y = \dfrac{1}{x + 4} - 1$

Lesson 5.2, p. 262

1. **a)** A; The function has a zero at 3 and the reciprocal function has a vertical asymptote at $x = 3$. The function is positive for $x < 3$ and negative for $x > 3$.
b) C; The function in the numerator factors to $(x + 3)(x - 3)$. $(x - 3)$ factors out of both the numerator and the denominator. The equation simplifies to $y = x + 3$, but has a hole at $x = 3$.
c) F; The function in the denominator has a zero at $x = -3$, so there is a vertical asymptote at $x = -3$. The function is always positive.
d) D; The function in the denominator has zeros at $y = 1$ and $y = -3$. The rational function has vertical asymptotes at $x = 1$ and $x = -3$.
e) B; The function has no zeros and no vertical asymptotes or holes.
f) E; The function in the denominator has a zero at $x = 3$ and the rational function has a vertical asymptote at $x = 3$. The degree of the numerator is exactly 1 more than the degree of the denominator, so the graph has an oblique asymptote.

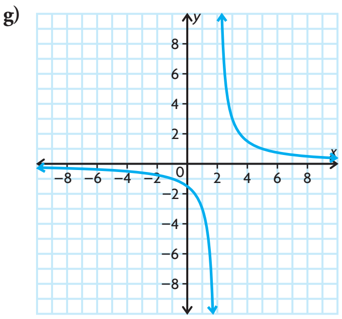
2. **a)** vertical asymptote at $x = -4$; horizontal asymptote at $y = 1$
b) vertical asymptote at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$
c) vertical asymptote at $x = 6$; horizontal asymptote at $y = 2$
d) hole at $x = -3$
e) vertical asymptotes at $x = -3$ and 5; horizontal asymptote at $y = 0$
f) vertical asymptote at $x = -1$; horizontal asymptote at $y = -1$
g) hole at $x = 2$
h) vertical asymptote at $x = \frac{5}{2}$; horizontal asymptote at $y = -2$
i) vertical asymptote at $x = -\frac{1}{4}$; horizontal asymptote at $y = 1$
j) vertical asymptote at $x = 4$; hole at $x = -4$; horizontal asymptote at $y = 0$
k) vertical asymptote at $x = \frac{3}{5}$; horizontal asymptote at $y = \frac{1}{5}$
l) vertical asymptote at $x = 4$; horizontal asymptote at $y = -\frac{3}{2}$

3. Answers may vary. For example:
a) $y = \dfrac{x - 1}{x^2 + x - 2}$
b) $y = \dfrac{1}{x^2 - 4}$
c) $y = \dfrac{x^2 - 4}{x^2 + 3x + 2}$
d) $y = \dfrac{2x}{x + 1}$
e) $y = \dfrac{x^3}{x^2 + 5}$

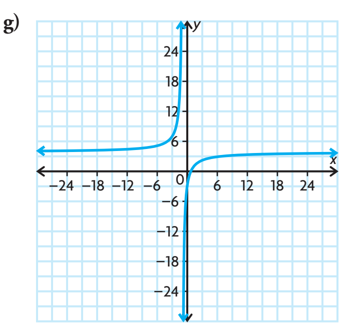
Lesson 5.3, pp. 272–274

1. **a)** A **b)** C **c)** D **d)** B

2. **a)** $x = 2$
b) As $x \to 2$ from the right, the values of $f(x)$ get larger. As $x \to 2$ from the left, the values become larger in magnitude but are negative.
c) $y = 0$
d) As $x \to -\infty$ and as $x \to \infty$, $f(x) \to 0$.
e) $D = \{x \in \mathbf{R} \mid x \neq 3\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
f) positive: $(2, \infty)$
negative: $(-\infty, 2)$
g)



3. **a)** $x = -1$
b) As $x \to -1$ from the left, $y \to \infty$. As $x \to -1$ from the right, $y \to -\infty$.
c) $y = 4$
d) As $x \to \pm\infty$, $f(x)$ gets closer and closer to 4.
e) $D = \{x \in \mathbf{R} \mid x \neq -1\}$
$R = \{y \in \mathbf{R} \mid y \neq 4\}$
f) positive: $(-\infty, -1)$ and $\left(\frac{3}{4}, -\infty\right)$
negative: $\left(-1, \frac{3}{4}\right)$
g)



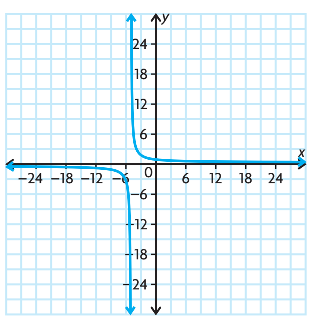
4. a) $x = -3$; As $x = -3$, $y = -\infty$ on the left. As $x = -3$, $y = \infty$ on the right.

b) $x = 5$; As $x = 5$, $y = -\infty$ on the left. As $x = 5$, $y = \infty$ on the right.

c) $x = \frac{1}{2}$; As $x = \frac{1}{2}$, $y = -\infty$ on the left. As $x = \frac{1}{2}$, $y = \infty$ on the right.

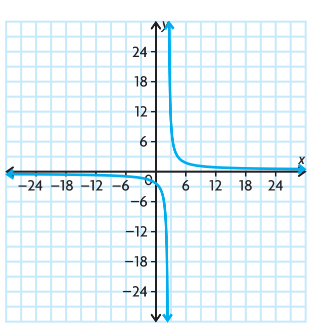
d) $x = -\frac{1}{4}$; As $x = -\frac{1}{4}$, $y = -\infty$ on the left. As $x = -\frac{1}{4}$, $y = \infty$ on the right.

5. a) vertical asymptote at $x = -5$
horizontal asymptote at $y = 0$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= \frac{3}{5}$
$f(x)$ is negative on $(-\infty, -5)$ and positive on $(-5, \infty)$.



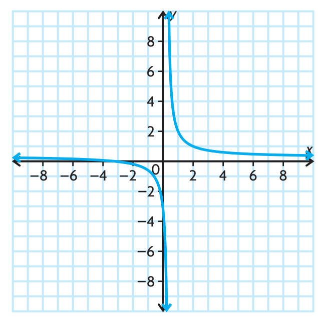
The function is decreasing on $(-\infty, -5)$ and on $(-5, \infty)$. The function is never increasing.

b) vertical asymptote at $x = \frac{5}{2}$
horizontal asymptote at $y = 0$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= -2$
$f(x)$ is negative on $\left(-\infty, \frac{5}{2}\right)$ and positive on $\left(\frac{5}{2}, \infty\right)$.



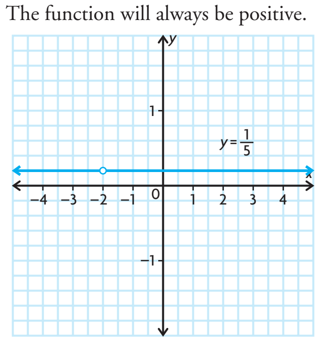
The function is decreasing on $\left(-\infty, \frac{5}{2}\right)$ and on $\left(\frac{5}{2}, \infty\right)$. The function is never increasing.

c) vertical asymptote at $x = \frac{1}{4}$
horizontal asymptote at $y = \frac{1}{4}$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{1}{4}\right\}$
$R = \left\{y \in \mathbf{R} \mid y \neq \frac{1}{4}\right\}$
x-intercept $= -5$
y-intercept $= -1$
$f(x)$ is positive on $(-\infty, -5)$ and $\left(\frac{1}{4}, \infty\right)$ and negative on $\left(-5, \frac{1}{4}\right)$.



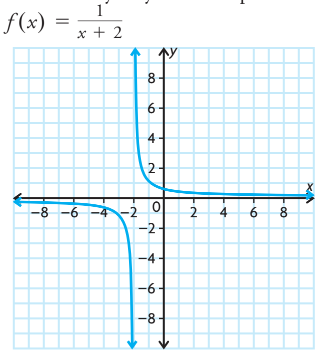
The function is decreasing on $\left(-\infty, \frac{1}{4}\right)$ and on $\left(\frac{1}{4}, \infty\right)$. The function is never increasing.

d) hole $x = -2$
$D = \{x \in \mathbf{R} \mid x \neq -2\}$
$R = \left\{y = \frac{1}{5}\right\}$
y-intercept $= \frac{1}{5}$
The function will always be positive.

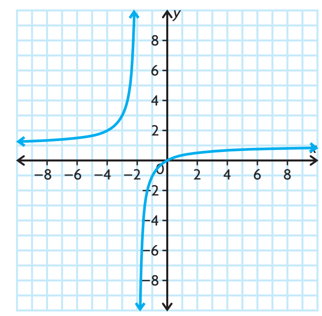


The function is neither increasing nor decreasing; it is constant.

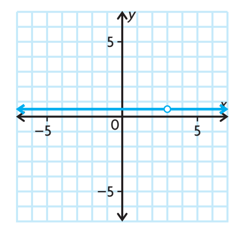
6. a) Answers may vary. For example:
$f(x) = \frac{1}{x + 2}$



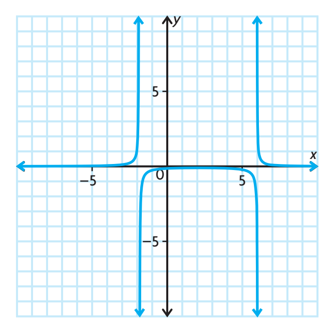
b) Answers may vary. For example:
$y = \frac{x}{x + 2}$



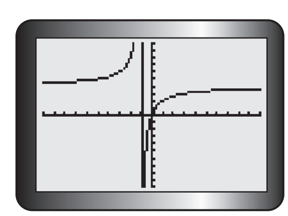
c) Answers may vary. For example:
$f(x) = \frac{x - 3}{2x - 6}$

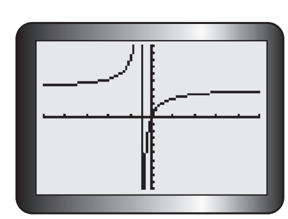


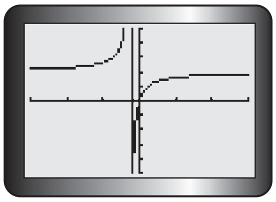
d) Answers may vary. For example:
$f(x) = \frac{1}{x^2 - 4x - 12}$

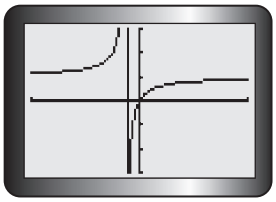


7. a)









The equation has a general vertical asymptote at $x = -\frac{1}{n}$. The function has a general horizontal asymptote at $y = \frac{8}{n}$. The vertical asymptotes are $-\frac{1}{8}$, $-\frac{1}{4}$, $-\frac{1}{2}$, and -1. The horizontal asymptotes are 8, 4, 2, and 1. The function contracts as n increases. The function is always increasing. The function is positive on $\left(-\infty, -\frac{17}{n}\right)$ and $\left(\frac{3}{10}, \infty\right)$. The function is negative on $\left(-\frac{17}{n}, \frac{3}{10}\right)$.

b) The horizontal and vertical asymptotes both approach 0 as the value of n increases; the x- and y-intercepts do not change, nor do the positive and negative characteristics or the increasing and decreasing characteristics.

c) The vertical asymptote becomes $x = \frac{17}{n}$ and the horizontal becomes $x = -\frac{10}{n}$. The function is always increasing. The function is positive on $\left(-\infty, \frac{3}{10}\right)$ and $\left(\frac{17}{n}, \infty\right)$. The function is negative on $\left(\frac{3}{10}, \frac{17}{n}\right)$. The rest of the characteristics do not change.

8. $f(x)$ will have a vertical asymptote at $x = 1$; $g(x)$ will have a vertical asymptote at $x = -\frac{3}{2}$. $f(x)$ will have a horizontal asymptote at $x = 3$; $g(x)$ will have a vertical asymptote at $x = \frac{1}{2}$.

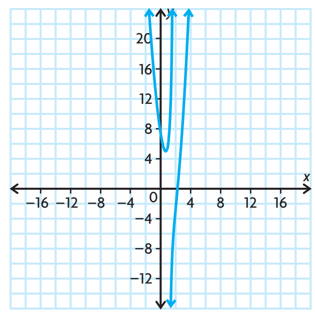
9. **a)** \$27 500
b) \$40 000
c) \$65 000
d) No, the value of the investment at $t = 0$ should be the original value invested.
e) The function is probably not accurate at very small values of t because as $t \to 0$ from the right, $x \to \infty$.
f) \$15 000

10. The concentration increases over the 24 h period and approaches approximately 1.89 mg/L.

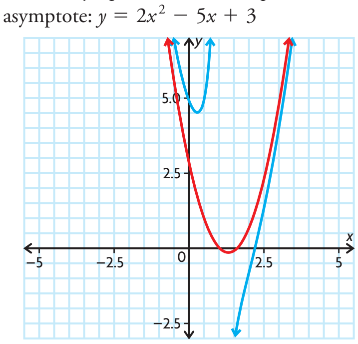
11. Answers may vary. For example, the rational functions will all have vertical asymptotes at $x = -\frac{d}{c}$. They will all have horizontal asymptotes at $y = \frac{a}{c}$. They will intersect the y-axis at $y = \frac{b}{d}$. The rational functions will have an x-intercept at $x = -\frac{b}{a}$.

12. Answers may vary. For example, $f(x) = \frac{2x^2}{2 + x}$.

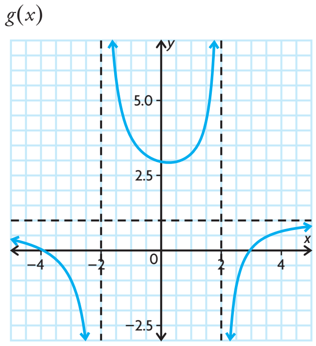
13. $f(x) = 2x^2 - 5x + 3 - \dfrac{2}{x - 1}$
As $x \to \pm\infty$, $f(x) \to \infty$.



vertical asymptote: $x = 1$; oblique asymptote: $y = 2x^2 - 5x + 3$



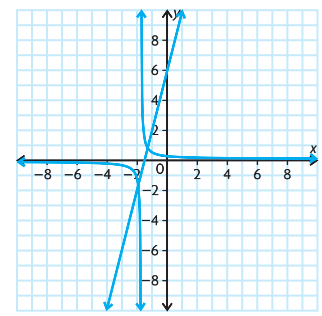
14. **a)** $f(x)$
b) $g(x)$ and $h(x)$
c) $g(x)$
d)



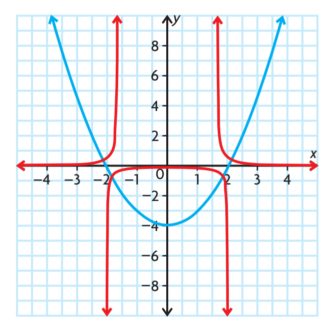
Mid-Chapter Review, p. 277

1. **a)** $\frac{1}{x - 3}$; $x = 3$
b) $\frac{1}{-4q + 6}$; $q = \frac{3}{2}$
c) $\frac{1}{z^2 + 4z - 5}$; $z = -5$ and 1
d) $\frac{1}{6d^2 + 7d - 3}$; $d = \frac{1}{3}$ and $-\frac{3}{2}$

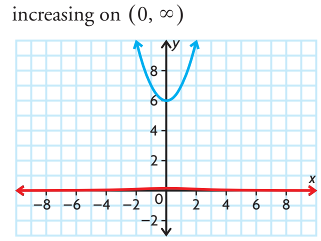
2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{x \in \mathbf{R}\}$; y-intercept $= 6$; x-intercept $= -\frac{3}{2}$; negative on $\left(-\infty, -\frac{3}{2}\right)$; positive on $\left(-\frac{3}{2}, \infty\right)$; increasing on $(-\infty, \infty)$



b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > -4\}$; y-intercept $= -4$; x-intercepts are 2 and -2; decreasing on $(-\infty, 0)$; increasing $(0, \infty)$; positive on $(-\infty, -2)$ and $(2, \infty)$; negative on $(-2, 2)$

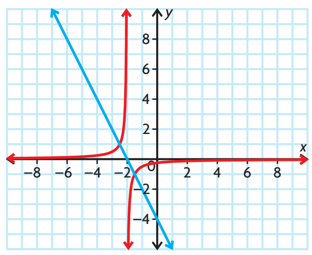


c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 6\}$; no x-intercepts; function will never be negative; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$



Answers

d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; x-intercept $= -2$; function is always decreasing; positive on $(-\infty, -2)$; negative on $(-2, \infty)$

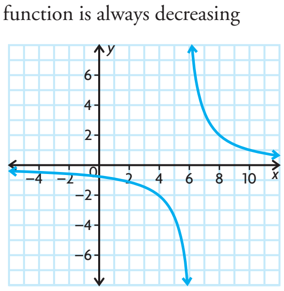


3. Answers may vary. For example: (1) Hole: Both the numerator and the denominator contain a common factor, resulting in $\frac{0}{0}$ for a specific value of x. (2) Vertical asymptote: A value of x causes the denominator of a rational function to be 0. (3) Horizontal asymptote: A horizontal asymptote is created by the ratio between the numerator and the denominator of a rational function as the function $\rightarrow \infty$ and $-\infty$. A continuous rational function is created when the denominator of the rational function has no zeros.

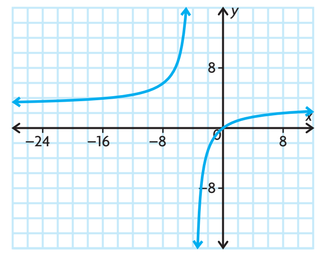
4. **a)** $x = 2$; vertical asymptote
b) hole at $x = 1$
c) $x = -\frac{1}{2}$; horizontal asymptote
d) $x = 6$; oblique asymptote
e) $x = -5$ and $x = 3$; vertical asymptotes

5. $y = \dfrac{x}{x-2}$, $y = 1$; $y = \dfrac{-7x}{4x+2}$, $y = \dfrac{-7}{4}$; $y = \dfrac{1}{x^2 + 2x - 15}$, $x = 0$

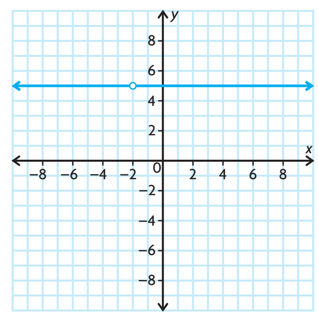
6. **a)** vertical asymptote: $x = 6$; horizontal asymptote: $y = 0$; no x-intercept; y-intercept: $-\frac{5}{6}$; negative when the denominator is negative; positive when the numerator is positive; $x - 6$ is negative on $x < 6$; $f(x)$ is negative on $(-\infty, 6)$ and positive on $(6, \infty)$; function is always decreasing



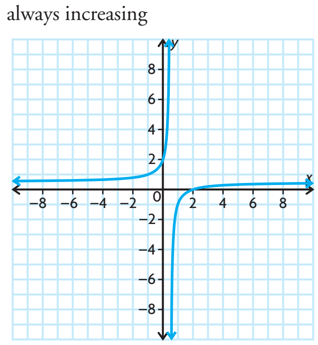
b) vertical asymptote: $x = -4$; horizontal asymptote: $y = 3$; x-intercept: $x = 0$; y-intercept: $f(0) = 0$; function is always increasing; positive on $(-\infty, -4)$ and $(0, \infty)$; negative on $(-4, 0)$



c) straight, horizontal line with a hole at $x = -2$; always positive and never increases or decreases



d) vertical asymptote: $x = \frac{1}{2}$; horizontal asymptote: $y = \frac{1}{2}$; x-intercept: $x = 2$; y-intercept: $f(0) = 5$; function is always increasing



7. Answers may vary. For example: Changing the function to $y = \frac{7x+6}{x+1}$ changes the graph. The function now has a vertical asymptote at $x = -1$ and still has a horizontal asymptote at $y = 7$. However, the function is now constantly increasing instead of decreasing. The new function still has an x-intercept at $x = -\frac{6}{7}$, but now has a y-intercept at $y = 6$.

8. $n = \frac{1}{3}$; $m = 35$

9. Answers may vary. For example, $f(x) = \frac{4x+8}{x+2}$.
The graph of the function will be a horizontal line at $y = 4$ with a hole at $x = -2$.

Lesson 5.4, pp. 285–287

1. 3; -2; Answers may vary. For example, substituting each value for x in the equation produces the same value on each side of the equation, so both are solutions.

2. **a)** $x = -3$ **b)** $x = 5$ **c)** $x = -1$ and 2 **d)** $x = -4$

3. **a)** $f(x) = \dfrac{x-3}{x+3} - 2$
b) $f(x) = \dfrac{3x-1}{x} - \dfrac{5}{2}$
c) $f(x) = \dfrac{x-1}{x} - \dfrac{x+1}{x+3}$
d) $f(x) = \dfrac{x-2}{x+3} - \dfrac{x-4}{x+5}$

4. **a)** $x = -9$ **b)** $x = 2$ **c)** $x = 3$ **d)** $x = -\frac{1}{2}$

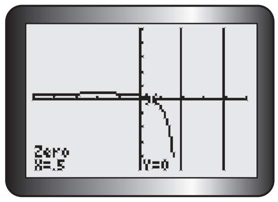
5. **a)** $x = 3$ **b)** $x = \frac{3}{4}$ **c)** $x = -9$ **d)** $x = 0$ **e)** $x = \frac{1}{4}$ **f)** $x = -23$

6. **a)** The function will have no real solutions.
b) $x = 3$ and $x = -0.5$
c) $x = -5$
d) $x = 0$ and $x = -1$
e) The original equation has no real solutions.
f) $x = 5$ and $x = 2$

7. **a)** $x = 6$ **b)** $x = 1.30, 7.70$ **c)** $x = 10$ **d)** $x = 3.25, 20.75$ **e)** $x = -1.71, 2.71$ **f)** $x = -0.62, 1.62$

8. **a)** $\dfrac{x+1}{x-2} = \dfrac{x+3}{x-4}$
Multiply both sides of the equation by the LCD, $(x-2)(x-4)$.
$$(x-2)(x-4)\left(\frac{x+1}{x-2}\right) = (x-2)(x-4)\left(\frac{x+3}{x-4}\right)$$
$(x-4)(x+1) = (x-2)(x+3)$
Simplify. $x^2 - 3x - 4 = x^2 + x - 6$
Simplify the equation so that 0 is on one side of the equation.
$$x^2 - x^2 - 3x - x - 4 + 6 = x^2 - x^2 + x - x - 6 + 6$$
$-4x + 2 = 0$
$-2(2x - 1) = 0$
Since the product is equal to 0, one of the factors must be equal to 0. It must be $2x - 1$ because 2 is a constant.
$$2x - 1 = 0$$
$$2x - 1 + 1 = 0 + 1$$
$$2x = 1$$
$$\frac{2x}{2} = \frac{1}{2}$$
$$x = \frac{1}{2}$$

b) $\dfrac{\frac{1}{2}+1}{\frac{1}{2}-2} = -1$ and $\dfrac{\frac{1}{2}+3}{\frac{1}{2}-4} = -1$

c) 

9. $w = 9.271$

10. Machine A $= 25.8$ min;
Machine B $= 35.8$ min

11. 75; \$4.00

12. **a)** After 6666.67 s
b) The function appears to approach $9\ \text{kg/m}^3$ as time increases.

13. **a)** Tom $= 4$ min; Carl $= 5$ min; Paco $= 2$ min
b) 6.4 min

14. Answers may vary. For example, you can use either algebra or graphing technology to solve a rational equation. With algebra, solving the equation takes more time, but you get an exact answer. With graphing technology, you can solve the equation quickly, but you do not always get an exact answer.

15. $x = -3.80, -1.42, 0.90, 4.33$

16. **a)** $x = 0.438$ and 1.712
b) $(0, 0.438)$ and $(1.712, \infty)$

Lesson 5.5, pp. 295–297

1. **a)** $(\infty, 1)$ and $(3, \infty)$
b) $(-0.5, 1)$ and $(2, \infty)$

2. **a)** Solve the inequality for x.

$$\frac{6x}{x+3} \le 4$$
$$\frac{6x}{x+3} - 4 \le 0$$
$$\frac{6x}{x+3} - 4\frac{x+3}{x+3} \le 0$$
$$\frac{6x-4x-12}{x+3} \le 0$$
$$\frac{2x-12}{x+3} \le 0$$
$$\frac{2(x-6)}{x+3} \le 0$$

b) 

c) $(-3, 6]$

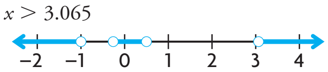
3. **a)**
$$x + 2 > \frac{15}{x}$$
$$x + 2 - \frac{15}{x} > 0$$
$$\frac{x^2}{x} + \frac{2x}{x} - \frac{15}{x} > 0$$
$$\frac{x^2+2x-15}{x} > 0$$
$$\frac{(x+5)(x-3)}{x} > 0$$

b) negative: $x < -5$ and $0 < x < 3$;
positive: $-5 < x < 0$, $x > 3$
c) $\{x \in \mathbf{R} \mid -5 < x < 0 \text{ or } x > 3\}$ or $(-5, 0)$ or $(3, \infty)$

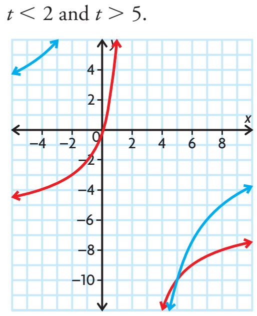
4. **a)** $5 < x < -4.5$
b) $-7 < x < -5$ and $x > -3$
c) $0 < x < 2$ and $x > 8$
d) $-6.8 \le x < -4$ and $x > 3$
e) $x < -1$ and $-\frac{1}{7} < x < 0$
f) $-1 < x < \frac{7}{8}$ and $x < 4$

5. **a)** $t < -3$ or $1 < t < 4$
b) $-3 \le t \le 2$ or $t > 4$
c) $-\frac{1}{2} < t < \frac{1}{3}$ or $t > \frac{1}{2}$
d) $t < -2$ and $-2 < t < 3$
e) $t < -5$ and $-2 < t < 0$
f) $-1 \le t < 0.25$ and $2 \le t < 9$

6. **a)** $x \in (-\infty, -6)$ or $x \in (-1, 4)$
b) $x \in (3, \infty)$
c) $x \in (-4, -2)$ or $x \in (-1, 2)$
d) $x \in (-\infty, -9)$ or $x \in [-3, -1)$ or $x \in [3, \infty)$
e) $x \in (-2, 0)$ or $x \in (4, \infty)$
f) $x \in (-\infty, -4)$ or $x \in (4, \infty)$

7. **a)** $x < -1$, $-0.2614 < x < 0.5$, $x > 3.065$
b) 

c) Interval notation: $(-\infty, -1)$, $(-0.2614, 0.5)$, $(3.065, \infty)$
Set notation: $\{x \in \mathbf{R} \mid x < -1, -0.2614 < x < 0.5, \text{ or } x > 3.065\}$

8. **a)** $t < 2$ and $t > 5$.
b) 

c) It would be difficult to find a situation that could be represented by these rational expressions because very few positive values of x yield a positive value of y.

9. The only values that make the expression greater than 0 are negative. Because the values of t have to be positive, the bacteria count in the tap water will never be greater than that of the pond water.

10. **a)** $\dfrac{(x^2 - 4x - 5)}{2x < 0}$
b)

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$(x-5)$	−	−	−	+
$(x+1)$	−	+	+	+
$2x$	−	−	+	+
$\frac{(x-5)(x+1)}{2x}$	−	+	−	+

The inequality is true for $x < -1$ and $0 < x < 5$

11. when $x > 5$

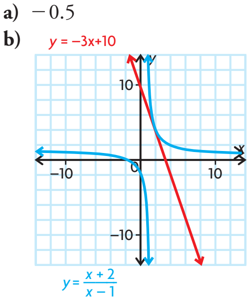
12. **a)** The first inequality can be manipulated algebraically to produce the second inequality.
b) Graph the equation $y = \frac{x+1}{x-1} - \frac{x+3}{x+2}$ and determine when it is negative.
c) The values that make the factors of the second inequality zero are -5, -2, and 1. Determine the sign of each factor in the intervals corresponding to the zeros. Determine when the entire expression is negative by examining the signs of the factors.

13. $[2, 4)$ and $(4, \infty)$

14. $14.48 < x < 165.52$ and $180 < x < 360$

15. $0 < x < 2$

Lesson 5.6, pp. 303–305

1. **a)** -0.5
b) 

slope $= -3$

2. -3

3. -3

4. -1

5. **a)** 0.01
b) -0.3
c) -1.3
d) 6

6. **a)** slope $= 286.1$; vertical asymptote: $x = -1.5$
b) slope $= -2.74$; vertical asymptote: $x = -5$
c) slope $= 44.65$; vertical asymptote: $x = -\frac{5}{3}$
d) slope $= -1.26$; vertical asymptote: $x = 6$

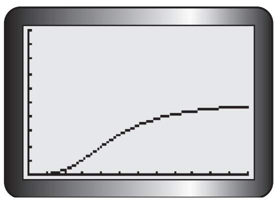
Answers

7. **a)** 0.01
b) 0.34

8. **a)** $R(x) = \dfrac{15x}{2x^2 + 11x + 5}$
b) 0.3, -0.03

9. **a)** \$5.67
b) -2

10. **a)** 68.46
b) 94.54
c)



The number of houses that were built increases slowly at first, but rises rapidly between the third and sixth months. During the last six months, the rate at which the houses were built decreases.

11. Answers may vary. For example: $14 \le x \le 15$; $x = 14.5$

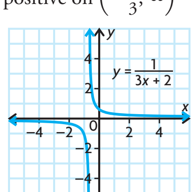
12. **a)** Find $s(0)$ and $s(6)$, and then solve $\dfrac{s(6) - s(0)}{6 - 0}$.
b) The average rate of change over this interval gives the object's speed.
c) To find the instantaneous rate of change at a specific point, you could find the slope of the line that is tangent to the function $s(t)$ at the specific point. You could also find the average rate of change on either side of the point for smaller and smaller intervals until it stabilizes to a constant. It is generally easier to find the instantaneous rate using a graph, but the second method is more accurate.
d) The instantaneous rate of change for a specific time, t, is the acceleration of the object at this time.

13. $y = -0.5x - 2.598$; $y = -0.5x + 2.598$; $y = 4x$

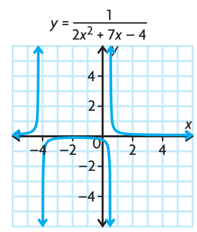
14. The instantaneous rate of change at $(0, 0) = 4$. The rate of change at this rate of change will be 0.

Chapter Review, pp. 308–309

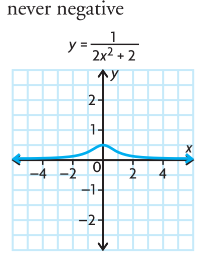
1. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; x-intercept $= -\frac{2}{3}$; y-intercept $= 2$; always increasing; negative on $\left(-\infty, -\frac{2}{3}\right)$; positive on $\left(-\frac{2}{3}, \infty\right)$



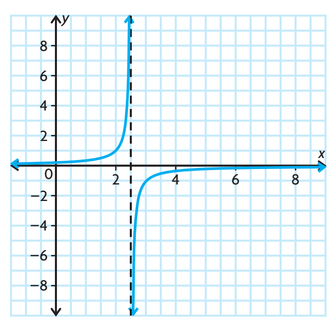
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > -10.125\}$; x-intercept $= 0.5$ and -4; positive on $(-\infty, -4)$ and $(0.5, \infty)$; negative on $(-4, 0.5)$; decreasing on $(-\infty, -10.125)$; increasing on $(-10.125, \infty)$



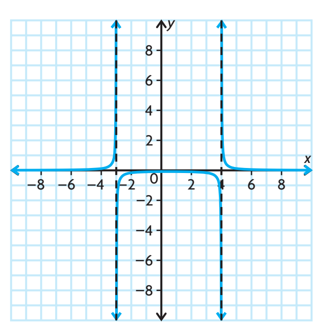
c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 2\}$; no x-intercepts; y-intercept $= 2$; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; always positive, never negative



2. **a)**



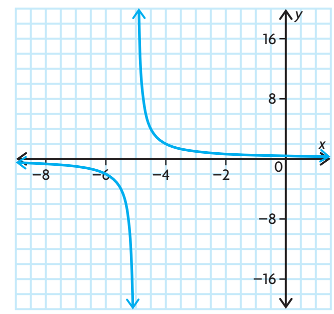
b)



3. **a)** $x = -17$
b) $x = -\frac{3}{5}$; horizontal asymptote; $y = \frac{2}{5}$
c) $x = 0.5$; hole at $x = -11$
d) $x = 1$; oblique asymptote; $y = 3x + 3$

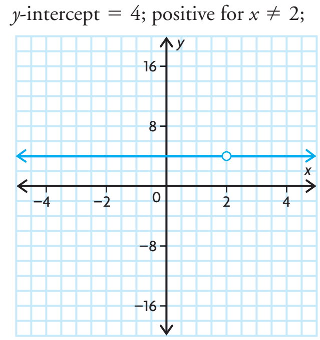
4. The locust population increased during the first 1.75 years, to reach a maximum of 1 248 000. The population gradually decreased until the end of the 50 years, when the population was 128 000.

5. **a)** x-intercept $= 2$: horizontal asymptote: $y = 0$; y-intercept $= \frac{2}{5}$: vertical asymptote: $x = -5$;



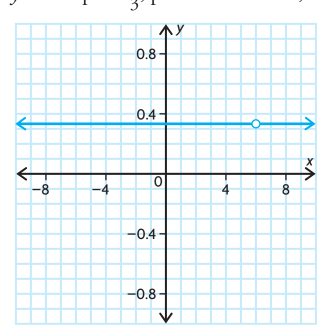
The function is never increasing and is decreasing on $(-\infty, -5)$ and $(-5, \infty)$. $D = \{x \in \mathbf{R} \mid x \ne -5\}$; negative for $x < -5$; positive for $x > -5$

b) $D = \{x \in \mathbf{R} \mid x \ne 2\}$; no x-intercept; y-intercept $= 4$; positive for $x \ne 2$;



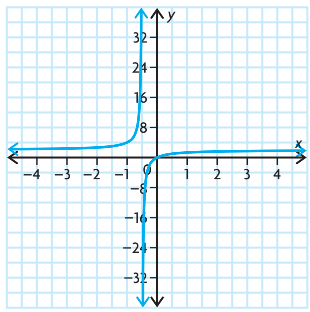
never increasing or decreasing

c) $D = \{x \in \mathbf{R} \mid x \ne 6\}$; no x-intercept; y-intercept $= \frac{1}{3}$; positive for $x \ne 6$;



never increasing or decreasing

d) $x = -0.5$; vertical asymptote: $x = -0.5$; $D = \{x \in \mathbf{R} \mid x \neq -0.5\}$; x-intercept $= 0$; y-intercept $= 0$; horizontal asymptote $= 2$; $R = \{y \in \mathbf{R} \mid x \neq 2\}$; positive on $x < -0.5$ and $x > 0$; negative on $-0.5 < x < 0$



The function is never decreasing and is increasing on $(-\infty, -0.5)$ and $(-0.5, \infty)$.

6. Answers may vary. For example, consider the function $f(x) = \frac{1}{x - 6}$. You know that the vertical asymptote would be $x = 6$. If you were to find the value of the function very close to $x = 6$ (say $f(5.99)$ or $f(6.01)$) you would be able to determine the behaviour of the function on either side of the asymptote.

$$f(5.99) = \frac{1}{(5.99) - 6} = -100$$

$$f(6.01) = \frac{1}{(6.01) - 6} = 100$$

To the left of the vertical asymptote, the function moves toward $-\infty$. To the right of the vertical asymptote, the function moves toward ∞.

7. **a)** $x = 6$
b) $x = 0.2$ and $x = -\frac{2}{3}$
c) $x = -6$ or $x = 2$
d) $x = -1$ and $x = 3$

8. about 12 min

9. $x = 1.82$ days and 3.297 days

10. **a)** $x < -3$ and $-2.873 < x < 4.873$
b) $-16 < x < -11$ and $-5 < x$
c) $-2 < x < -1.33$ and $-1 < x < 0$
d) $0 < x < 1.5$

11. $-0.7261 < t < 0$ and $t > 64.73$

12. **a)** -6; $x = 3$
b) 0.2; $x = -2$ and $x = -1$

13. **a)** 0.455 mg/L/h
b) -0.04 mg/L/h
c) The concentration of the drug in the blood stream appears to be increasing most rapidly during the first hour and a half; the graph is steep and increasing during this time.

14. $x = 5$ and $x = 8$; $x = 6.5$

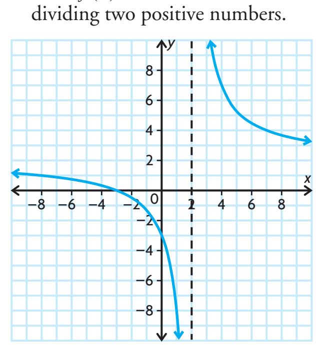
15. **a)** As the x-coordinate approaches the vertical asymptote of a rational function, the line tangent to graph will get closer and closer to being a vertical line. This means that the slope of the line tangent to the graph will get larger and larger, approaching positive or negative infinity depending on the function, as x gets closer to the vertical asymptote.
b) As the x-coordinate grows larger and larger in either direction, the line tangent to the graph will get closer and closer to being a horizontal line. This means that the slope of the line tangent to the graph will always approach zero as x gets larger and larger.

Chapter Self-Test, p. 310

1. **a)** B
b) A

2. **a)** If $f(n)$ is very large, then that would make $\frac{1}{f(n)}$ a very small fraction.
b) If $f(n)$ is very small (less than 1), then that would make $\frac{1}{f(n)}$ very large.
c) If $f(n) = 0$, then that would make $\frac{1}{f(n)}$ undefined at that point because you cannot divide by 0.
d) If $f(n)$ is positive, then that would make $\frac{1}{f(n)}$ also positive because you are dividing two positive numbers.

3.



4. 4326 kg; \$0.52/kg

5. **a)** Algebraic; $x = -1$ and $x = -3$
b) Algebraic with factor table
The inequality is true on $(-10, -5.5)$ and on $(-5, 1.2)$.

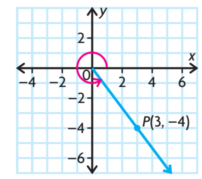
6. **a)** To find the vertical asymptotes of the function, find the zeros of the expression in the denominator. To find the equation of the horizontal asymptotes, divide the first two terms of the expressions in the numerator and denominator.
b) This type of function will have a hole when both the numerator and the denominator share the same factor $(x + a)$.

Chapter 6

Getting Started, p. 314

1. **a)** 28°
b) 332°

2. **a)**



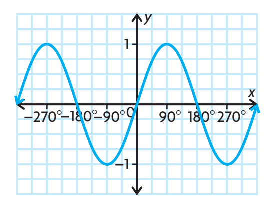
$\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,
$\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

b) 307°

3. **a)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **e)** $-\sqrt{2}$
b) 0 **d)** $\frac{1}{2}$ **f)** -1

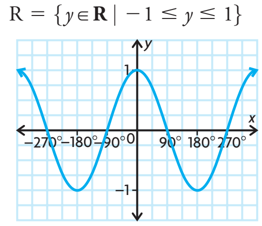
4. **a)** 60°, 300°
b) 30°, 210°
c) 45°, 225°
d) 180°
e) 135°, 315°
f) 90°

5. **a)**



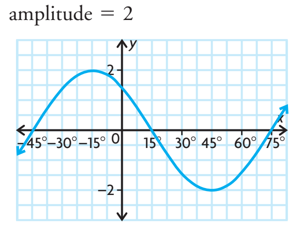
period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

b)



period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

6. **a)** period $= 120°$; $y = 0$; 45° to the left; amplitude $= 2$



Answers